The Dining Bidder Problem:
à la russe et à la française

RENATO PAES LEME, VASILIS SYRGKANIS and ÉVA TARDOS
Cornell University

Item bidding auctions are a line of research which provides a simple and often efficient alternative to traditional combinatorial auction design - in particular, they were inspired by real world auction houses, like eBay and Sotheby's. We survey the literature from a culinary perspective, offering an intuitive illustration of the welfare in simultaneous and sequential auctions. Welfare in simultaneous first and second price auctions is high when bidders have complement-free valuations. In contrast, sequential second price auctions can lead to bad outcomes due to signaling problems and even in the case of first price, a good outcome is only guaranteed for unit demand bidders. We give an intuitive interpretation of an example with bad welfare in sequential first price auctions with submodular bidders from Paes Leme, Syrgkanis and Tardos (SODA'12).

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> "Today, dishes are served one after the other, so their order of consumption, whatever it may be, is clear. Such was not the case for the *service à la française* meal etiquette that prevailed until the mid-nineteenth century [...] composed of several dishes brought to the table at the same time." (in "Arranging the Meal: A History of Table Service in France" [Flandrin 2007])

The *service à la française*, with all its dishes brought all at once was impressive, but also impractical, and so was soon substituted in most Western countries by the *service à la russe* in which the dishes are brought to the table sequentially.

This naturally motivates the following game theory question: consider a dinner with n guests, where the restaurant decides to hold an auction for each dish instead of selling it for a fixed-price. In this letter we consider the efficiency of such an auction under the *service à la française* and under the *service à la russe*. We consider two choices by the restaurant: (i) how to serve the dishes (simultaneously or sequentially) and (ii) which auction to run for each dish (first or second price).

It is important to stress that the guests will derive a combinatorial satisfaction for each subset of dishes and each guest has a different and private preference. For example, one guest might want either oysters or caviar but derive no additional benefit if he has both of them. In this case, we say those are *perfect substitutes*. We say that the satisfaction of a guest is *submodular* if the marginal satisfaction for one extra dish is smaller for a larger meal than for a smaller meal. For example, the extra satisfaction of eating artichokes after bœuf bourguignon is larger then the extra satisfaction of eating artichokes after bœuf bourguignon and a bouillabaisse.

We say that the guest is *easily satiated* (his satisfaction is *unit demand*) if the guest gets satiated so quickly that if he gets more then one dish, he will only eat his favorite one, derive satisfaction from it and ignore the rest. One can define more classes of satisfaction functions for the guests: *subadditive, fractionally subadditive (xos), walrasian, gross substitutes* ... - which are too many to be discussed in the letter, after all, there at too many ways in which food can be appreciated. We refer to [Lehmann et al. 2006] and [Feige 2006] for an overview.

For each design the restaurant chooses and for each class of satisfaction functions of the guests, two questions can be asked: (i) when does equilibrium exist – i.e. do they ever agree and get to eat and (ii) what is the overall sum of satisfaction of the guests (not counting payments, that take away from the satisfaction of the guest, but contribute to the business) compared to the optimum – which in the end is all the chef cares about. This last quantity is usually referred to as *welfare*.

Service à la française. Without further ado, we begin to ~~savor~~ survey the literature, which followed the chronological development in the French cuisine, i.e., started by analyzing simultaneous bids. Christodoulou, Kovács and Schapira [2008] first studied the à la française service where all the guests have submodular satisfaction and each dish is auctioned using a second price auction. They showed that whenever a Nash equilibrium of this game exists, it is a 2-approximation of the optimum. Surprisingly, their result holds even in the Bayesian setting, i.e., where each guest has Bayesian uncertainty about the preferences of the other guests. This result was subsequently improved by Bhawalkar and Roughgarden [2011] who extend the 2-approximation to guests with subadditive satisfaction functions for the case where guests know each other's preferences. In the case where guests are uncertain about each other, Feldman, Fu, Gravin and Lucier [2012] show 4-approximation (improving on the previously known logarithmic bound in [Bhawalkar and Roughgarden 2011]).

Hassidim, Kaplan, Mansour and Nisan [2011] analyze the à la française service with a twist in the design: each dish is now sold using a first-price auction. Bikchandani [1999] showed that pure equilibria are always optimal – which definitely pleases the chef – however, a pure equilibrium only exists if the satisfaction functions of the guests are *walrasian*. In order to cope with the absence of pure equilibria, [Hassidim et al. 2011] analyze mixed Nash equilibria, in which the guests use randomness (e.g. by tossing a toast with butter on one side) to decide their bids. They show that such mixed equilibria are a 2-approximation of the optimum for *submodular* satisfaction functions and $O(\log m)$ for *subadditive* satisfaction. Syrgkanis [2012] improved the bound for submodular satisfaction functions to $e/(e-1) \approx 1.58$ and Feldman et al. [2012] improved the bound for subadditive functions to 2. These bounds carry over to the Bayesian setting.

Service à la russe. In Paes Leme, Syrgkanis and Tardos [2012] we propose a rigorous analysis of the à la russe service. Dishes are brought to the table one at a time and guests bid. We assume perfectly-logical guests who can forward reason and see the impact of their actions in the future. In other words, guests play a *subgame perfect equilibrium.* As an example, Alice might think that if she wins the

oysters now, Bob will want the caviar and won't want the thick soup. So, even if Alice doesn't want the caviar, she might want to bid for it so that she can later win the thick soup. Dinner gets quite complicated, but one good piece of news is that unlike the previous models, a pure equilibrium *always* exists, regardless of the satisfaction functions and both for first and second price auctions.

The first bad news is that if the auction run for each dish is a second price auction, the overall welfare can be arbitrarily low compared to the optimum, even if the guests are *real gluttons*. By *real gluttons* we mean that their satisfaction for a set of dishes is the exact sum of their satisfactions for each dish individually. *Real gluttons* are usually called *additive* bidders. The reason is that second price allows the guests to engage in very non-trivial signaling behavior through their bids.

Using first-price solves the signaling problem and generates much more well behaved dinner-auctions. In fact, if all the guests are *real gluttons* (*additive*), the outcome is always optimal, and Paes Leme, Syrgkanis and Tardos [2012] show that if the guests are *easily satiated* (*unit-demand*), the outcome is a 2-approximation of the optimum if the players know each other's satisfaction function. When guests are *unit-demand*, but have uncertainty about each other satisfaction functions, Syrgkanis and Tardos [2012] showed the outcome is a 3.16-approximation.

Motivated by the results on the service à la française, it was expected that if guests were *submodular*, the outcome would be a constant approximation of the optimum in the service à la russe as well. It came to us as a surprise that the sequential game can lead to an equilibrium that is arbitrarily worse then the optimal.

To close this letter, we give an informal illustration of an example from Paes Leme, Syrgkanis and Tardos [2012] of how sequential rationality of the guests can lead to a very inefficient dinner. We begin by recalling that the final satisfaction of each guest is her satisfaction for the dishes she got minus the amount she paid (let us assume her satisfaction is measured in dollars).

Too much rationality destroys your dinner

Consider k pieces of bread, one fish dish (thon à la provençale), one beef dish (bœuf à la catalane) and one pork dish (roti de pôrc). The dishes are going to be brought to the table and auctioned in the order mentioned. There are four guests: Alice, Bob, Charlie and Danah. Alice and Bob are *real gluttons* (i.e. *additive*). Alice's satisfaction is 1 for each piece of bread, 2 for the beef and zero for the rest. Bob's satisfaction is 1 for each piece of bread, 2 for the pork and zero for the rest. Charlie's satisfaction is ϵ for each piece of bread and $4 - \frac{\epsilon}{2} - \epsilon b_c$ for the fish, where b_c is the pieces of bread he gets; the more bread he eats, the less he wants the fish.

Enters Danah, the troublemaker. She has a complicated satisfaction function: essentially, she derives satisfaction 2 from the beef and the pork, but this satisfaction decreases slightly the more bread she eats. Also, it becomes almost zero if she eats the fish before. More precisely, her value is ϵ for each piece of bread, 4 for the fish and $M(b_d, f_d) = 2(1 - f_d) + \frac{\epsilon}{2}(k - b_d)$ for the beef and the same for the pork, where b_d is the number of pieces of bread she ate and f_d is 1 if she ate the fish and zero otherwise; bread decreases Danah's appetite for the heavier meat but not for the lighter fish.

	bread$_1$	$\cdots$	bread$_k$	fish	beef	pork
A	1	$\cdots$	1		2	
B	1	$\cdots$	1			2
C	ϵ	$\cdots$	ϵ	$4 - \frac{\epsilon}{2} - \epsilon b_c$		
D	ϵ	$\cdots$	ϵ	4	$M(b_d, f_d)$	$M(b_d, f_d)$

The optimum welfare is created when either Bob or Alice gets the bread, Charlie gets the fish and Danah gets the beef and the pork. The total welfare in this case is $k + 8$ (ignoring the ϵ terms)[1]. However, since Alice and Bob both desire the bread equally, neither of them will ever get it for a price less than one, so they will never derive net benefit from it. In equilibrium, Alice and Bob can do something much smarter: both of them leave the bread and let Danah acquire it for zero price. Danah eats all the bread and gets full, decreasing her value for the beef and pork. Now, given that she ate too much bread, she will much prefer to compete for the fish with Charlie rather then competing for the beef and the pork with Alice and Bob.[2] This way, Danah gets the fish and therefore she has essentially no additional value for the rest of the dishes. Now, Alice can get the beef and Bob can get the pork - both almost for free. All of them feast on their meals (except poor Charlie who did not get anything), but now the overall welfare is 8. As k grows, the gap between the optimal welfare and the welfare in equilibrium gets arbitrarily large.

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[1]There exist walrasian prices to support this allocation, although values are not gross-substitutes.
[2]Danah's logic: "if I don't get the fish I will derive utility $\epsilon(k - b_d)$ combined from the meat. So I will bid $4 - \epsilon(k - b_d)$ on the fish." Danah wins the fish iff $b_d + b_c = k$, i.e. either she or Charlie ate all the bread. Thus it is in Alice and Bob's best interest to avoid eating any of the bread.