Editor's Puzzle: Combinatorial Auction Winner Determination

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This puzzle asks you to solve an instance of the winner determination problem. Of course, this small instance can easily be solved by computer, and you are allowed to do so. However, to solve the puzzle completely, you must provide not only the efficient allocation, but also a (nice) proof of optimality. (Hint: look for a pattern in the bids.)

Solutions should be sent to the editor at `conitzer@cs.duke.edu` with subject header **SIGecom Exchanges Puzzle**. The author of the most elegant solution (as judged by the editor) will be allowed to publish his or her proof in the next issue of the Exchanges (ties will be broken towards earlier submissions). To make the solution accessible to a wide audience, try to minimize technical jargon in the proof. The editor will not give any feedback on submitted solutions and ignore any requests for hints, *etc.*

In this combinatorial auction, there are 5 items, A, B, C, D, E, and 12 (single-minded) bids:

(1) \$5 for $\{A, B\}$,
(2) \$10 for $\{A, C\}$,
(3) \$24 for $\{A, C, D\}$,
(4) \$51 for $\{A, C, D, E\}$,
(5) \$13 for $\{B, C\}$,
(6) \$27 for $\{B, C, D\}$,
(7) \$43 for $\{B, D, E\}$,
(8) \$29 for $\{B, E\}$,
(9) \$25 for $\{C, D\}$,
(10) \$48 for $\{C, D, E\}$,
(11) \$14 for $\{D\}$,
(12) \$23 for $\{E\}$.

You are asked to solve the standard winner determination problem: label each bid as accepted or rejected to maximize the total value of the accepted bids, under

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the constraint that each item is allocated at most once. Bids must be accepted in full or not at all; VCG payments, *etc.* need not be computed. But you must give a proof of optimality that is as elegant as possible.