Ex-Post Equilibria in Combinatorial Auctions

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Given a game with incomplete information, such as a combinatorial auction, one may ask what is a good solution concept for (or predictor for players' behaviors in) such a game. Most of the literature on combinatorial auctions appeals to a non-Bayesian setting, and to the concept of dominant strategy implementation. A nice property of a dominant strategy for a given game is its uniqueness. However, this nice property and the natural appeal of dominant strategies might be misleading.

In a general non-Bayesian setting, one can define the natural solution concept of ex-post equilibrium. Roughly speaking, an ex-post equilibrium is a strategy profile in which unilateral deviations are not beneficial regardless of the state of nature. Notice that in difference to a dominant strategy, where arbitrary behaviors of the other players are considered, in an ex-post equilibrium only other players' behaviors which conform to the prescribed strategy profile are considered. From a purely economic mechanism design perspective one may be tempted to ignore ex-post equilibria: The revelation principle implies that, in a private value setting, if a function is implemented as an ex-post equilibrium, then it is also implementable as a dominant strategy equilibrium of another mechanism. However, in a computational setting the revelation principle may be of little use: the translation from a mechanism to a corresponding revelation mechanism may be exponential. Hence, one should carefully characterize the set of ex-post equilibria of a game with incomplete information even if this game possesses a dominant strategy equilibrium; in fact, in principle, even if a game possesses a dominant strategy equilibrium, there can be another ex-post equilibrium which will be selected due to the fact that this equilibrium is more tractable from the computational perspective than the dominant strategy equilibrium.

The above argument is not hypothetical. Indeed, as it turns out, it touches on the most famous mechanism in the context of economic mechanism design in general, and combinatorial auctions in particular: the VCG combinatorial auction. While the literature on the VCG mechanism acknowledged the existence of a variety of ad-hoc equilibria for particular valuations, when considering the general setting the literature refers to the VCG mechanism as possessing a dominant strategy and does not refer to other non-Bayesian equilibria. However, as shown in [1]

such equilibria exist: Consider the situation where all players report truthfully their valuations for the set of all goods, and 0 otherwise; this strategy profile will be an ex-post equilibrium of the VCG mechanism. Needless to say, this ex-post equilibrium will have low communication complexity. Moreover, there are many other ex-post equilibria of the VCG mechanism. In [1] we define a whole set of such equilibria, termed *Bundling Equilibria*. In addition, a followup paper [2] shows that the bundling equilibria are the only ex-post equilibria of the VCG mechanism.

The above allows us to tackle a major puzzle. Assume we accept the point of view that players follow behaviors which are in equilibrium. The VCG mechanism possesses a dominant strategy equilibrium, but this equilibrium behavior is impossible to follow given the communication/computational constraints. So, how would the players' behave? The existence of ex-post equilibria of small communication complexity suggests an appropriate answer.

The reader should notice that the above perspective differs from the one used in the literature on resource-bounded reasoning. For example, in the context of the finitely repeated prisoners dilemma one can get cooperation in equilibrium when players are bounded machines, but this can not be obtained without the related constraint on the players' capabilities. On the other hand, in the context of the VCG mechanism the ex-post equilibria of low communication complexity are equilibria of the original mechanism, and are not a result of any exogenous assumption. Hence, the study of ex-post equilibria of combinatorial auctions illustrates an interesting format of bounded reasoning: Different ex-post equilibria of a given game may require different capabilities in order to be followed by the players; as a result, some ex-post equilibria of the original mechanism may be selected due to the fact they require a smaller amount of resources.

The above suggests a general lesson and direction one should bare in mind. The CS perspective suggests the need to consider non-Bayesian settings and non-revelation mechanisms; as a result the properties of all ex-post equilibria of a mechanism should be carefully considered, even in settings where dominant strategies exist. Unfortunately, this has not been the usual practice, as suggested by the VCG lesson. Detailed analysis of all ex-post equilibria may expose the effects of different communication/computation constraints on equilibrium selection given a suggested mechanism, and on the complexity of playing an equilibrium strategy in that setting.

REFERENCES

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