Editor's Puzzle: Product Adoption in a Social Network

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Solutions should be sent to the editor at `conitzer@cs.duke.edu` with subject header **SIGecom Exchanges Puzzle**. The author(s) of the most elegant solution (as judged by the editor) will be allowed to publish his or her or their proof in the next issue of the Exchanges (ties broken towards earlier submissions). To make the solution accessible to a wide audience, try to minimize technical jargon in the proof. The editor will not give any feedback on submitted solutions and ignore any requests for hints, *etc.*

We have n agents who, in sequence, have to decide whether to adopt technology A or technology B. The decision is irreversible. Each agent has a single other agent that she admires, drawn uniformly at random from the other agents. (Note that if x admires y, it is not necessarily the case that y admires x.) With probability p_0, an agent is a "B fanatic" who will certainly adopt B. With probability p_1, an agent is a "B copycat" who will adopt B if the agent that she admires has already adopted B, and A otherwise. With probability $1 - p_0 - p_1$, an agent is an "A fanatic" who will certainly adopt A. All random draws are independent.

Assuming that n is large, give a simple formula for the final fraction of agents who adopt B.

Hint: In a simulation with $n = 1000000$, $p_0 = .5$, and $p_1 = .5$, repeated 1000 times, the average final fraction of adopters was 0.6487, approximately $\sqrt{e} - 1$. In a simulation with $n = 1000000$, $p_0 = .1$, and $p_1 = .1$, repeated 1000 times, the average final fraction of adopters was 0.1052. In a simulation with $n = 1000000$, $p_0 = .1$, and $p_1 = .2$, repeated 1000 times, the average final fraction of adopters was 0.1107.

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