Simple versus Optimal Mechanisms

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1. INTRODUCTION

A striking theme in the theory of single-item auctions is that *simple auctions are optimal*. Foremost, Myerson [Myerson 1981] showed that practically prevalent reserve-price-based auctions are indeed expected revenue-maximizing in natural, though stylized, models. This is fortunate as these auctions are simple and easy to optimize (just set the reserve price). That practitioners widely employ reserve-price-based auctions — in settings much more complex that those where they are provably optimal — motivates our first question: *When are reserve-price-based mechanisms approximately optimal?*

Bulow and Klemperer [Bulow and Klemperer 1996] up the ante further by showing that *the reserve price is unnecessary*, in the sense that a seller of a single item earns more revenue from the Vickrey auction with one extra bidder than from the optimal auction with the original bidders. This guarantee for an auction even simpler than Vickrey with an optimal reserve motivates our second question: *How generally does a Bulow-Klemperer-style result approximately hold?*

Our recent paper [Hartline and Roughgarden 2009] provides tight approximation guarantees for the expected revenue of simple auctions, relative to that of an optimal auction, in environments much more general than single- or multi-unit auctions with i.i.d. bidders. The main message of the paper is that, in quite general settings, *simple auctions* — like the VCG mechanism supplemented with an anonymous reserve price, or with bidder-specific reserve prices, or with no reserves but in an expanded market — *provably approximate the optimal expected revenue to within a small constant factor.*

2. THE VCG MECHANISM WITH MONOPOLY RESERVE PRICES

We consider single-parameter environments, where each agent has a valuation for receiving service and there is a set system specifying *feasible sets*, i.e., sets of agents that can be served simultaneously. We assume that subsets of feasible sets are again feasible. Examples of such environments include multi-unit auctions with unit-demand (e.g., where feasible sets are those with cardinality at most k); and combinatorial auctions with single-minded bidders (where feasible sets are bidders seeking mutually disjoint bundles of goods).

Assuming that each agent's valuation is drawn independently from a known dis-

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tribution, Myerson [Myerson 1981] characterized optimal auctions for all single-parameter environments. In a simple environment like a k-item auction with i.i.d. bidder valuations (satisfying mild technical conditions), the optimal auction is simply the Vickrey auction with a reserve equal to the monopoly price (the revenue-optimal price for the trivial single-agent case). In more complex environments, however, the optimal auction is heavily dependent on the exact form of the bidders' valuation distribution(s). Is there a near-optimal auction that is far simpler?

> For every single-parameter environment and independent (not necessarily identical) valuation distributions that satisfy a standard hazard rate condition, the expected revenue of the VCG mechanism with monopoly reserve prices is at least 50% of that of an optimal mechanism.
>
> [Hartline and Roughgarden 2009, Theorem 3.2]

In more detail, the VCG mechanism with (bidder-specific) reserve prices $\mathbf{r} = (r_1, \ldots, r_n)$ is the following (truthful) direct-revelation mechanism: (1) every bidder i whose bid b_i is below the reserve r_i is discarded; (2) the VCG mechanism is invoked on the remaining bidders, which (assuming truthful bids) selects a feasible subset of them with maximum-possible welfare; and (3) each winning bidder i is charged the larger of its VCG payment in Step 2 and its reserve r_i. Recall that a monopoly price for a bidder with valuation drawn from the distribution F_i is a price that maximizes $p \cdot (1 - F_i(p))$.

We also prove that the factor of 50% is tight in the worst case, and that the hazard rate assumption is necessary for any constant-factor approximation guarantee. On the other hand, we can weaken the hazard rate assumption to "regularity" (see Myerson [Myerson 1981] for a definition) if we impose additional structure on the feasible sets [Hartline and Roughgarden 2009, Theorem 3.7].

3. BULOW-KLEMPERER-TYPE RESULTS

As already mentioned, Bulow and Klemperer [Bulow and Klemperer 1996] proved that, for a single-item auction with i.i.d. bidder valuations, the Vickrey auction with an additional bidder outperforms the optimal auction without an additional bidder (under mild distributional assumptions). For a general single-parameter environment, we consider duplicating every bidder and running the VCG mechanism. Each bidder and its duplicate have i.i.d. valuations, are interchangeable within the environment, and cannot be served simultaneously.[1] To what extent does the Bulow-Klemperer theorem continue to hold?

> For every single-parameter environment and independent (not necessarily identical) valuation distributions that satisfy a standard hazard rate condition, the expected revenue of the VCG mechanism with duplicate bidders (but no reserve prices) is at least 33% of that of an optimal mechanism in the original environment.
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> [Hartline and Roughgarden 2009, Theorem 4.2]

[1]Notice that for single-item auctions this is slighly different from the setting of Bulow-Klemperer, where a single duplicate bidder is interchangeable with any of the originals. When the bidders' distributions or the set system is asymmetric, our one-for-one interchangeability seems natural.

As before, we prove that the factor of 33% is tight in the worst case, and that the hazard rate assumption is necessary for any constant-factor approximation guarantee. Once again, we can weaken the hazard rate assumption on the valuation distributions to regularity (and also increase the approximation factor from 33% to 50%) if in exchange we assume additional structure on the feasible sets [Hartline and Roughgarden 2009, Theorem 4.4]. This improved bound is not known to be tight.

4. APPROXIMATION GUARANTEES FOR ANONYMOUS RESERVE PRICES

In an eBay auction, the seller is forced to choose an anonymous reserve price. If the seller has information to distinguish the probable valuations of different bidders — based on past history, say — then such an auction is not generally revenue-maximizing. Can near-optimal expected revenue still be achieved with an anonymous reserve price? Our Bulow-Klemperer-type results yield an affirmative answer as a corollary.

> For every single-item auction with independent (not necessarily identical) valuation distributions that satisfy a regularity condition, the expected revenue of the Vickrey auction with a suitable anonymous reserve price is at least 25% of that of an optimal auction.
>
> [Hartline and Roughgarden 2009, Theorem 5.1]

There are single-item environments for which every anonymous reserve price guarantees no more than 50% of the maximum-possible expected revenue. Determining the best-possible approximation guarantee achievable via an anonymous reserve price with non-identical bidders in single-item auctions remains an interesting open question.

REFERENCES

Bulow, J. and Klemperer, P. 1996. Auctions versus negotiations. *American Economic Review 86,* 1, 180–194.

Hartline, J. D. and Roughgarden, T. 2009. Simple versus optimal mechanisms. In *Proc. 11th ACM Conf. on Electronic Commerce (EC).*

Myerson, R. 1981. Optimal auction design. *Mathematics of Operations Research 6,* 1, 58–73.