Editor's Puzzle: Borrowing as Cheaply as Possible

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Solutions should be sent to the editor at `conitzer@cs.duke.edu` with subject header **SIGecom Exchanges Puzzle**. The author(s) of the most elegant solution (as judged by the editor) will be allowed to publish his or her or their proof in the next issue of the Exchanges (ties broken towards earlier submissions). To make the solution accessible to a wide audience, try to minimize technical jargon in the proof. Until the winner is chosen, the editor will not give any feedback on submitted solutions and ignore any requests for hints, *etc.*

Zoe needs to borrow $1000 from other people and is determined to do so as cheaply as possible. The various people involved each have different amounts of their own money that they can lend, and are willing to lend only to certain other people; for each of the people they are willing to lend to, they have a certain profit that they expect and, in some cases, a maximum amount they are willing to lend to that person. Specifically:

—Alice has $400. Alice is willing to lend to Denise, but wants a profit of $3 for every $100 she lends her. Alice is also willing to lend to Ed, but wants a profit of $4 for every $100 she lends him. Finally, Alice is willing to lend to Grace, but wants a profit of $7 for every $100 she lends her.

—Denise has $200. Denise is willing to lend to Bob, but wants a profit of $1 for every $100 she lends him. Denise is also willing to lend to Harry, but wants a profit of $4 for every $100 she lends him. Finally, Denise is willing to lend to Zoe; she expects no profit (but also no loss) for lending to Zoe, but she will lend at most $100 to Zoe.

Note that this allows for constructions such as the following. Alice lends all her $400 to Denise, expecting an additional $12 in return. Denise then lends all her $600 to Harry. For the $200 that was originally her own, Denise expects an additional $8 from Harry; for the $400 that Denise borrowed from Alice, she expects $28 from Harry, $12 of which is to pay off Alice, and the remaining $16 is Denise's required profit from lending $400 to Harry. Hence, Harry needs to pay an additional $36 in total. Another way to see this is that he is paying the $12 to Alice as well as $24 for borrowing $600 from Denise.

—Bob has $400. Bob is willing to lend to Ed, but wants a profit of $3 for every $100 he lends him. Bob is also willing to lend to Grace, but wants a profit of $5 for every $100 he lends her. Finally, Bob is willing to lend to Harry, but wants a profit of $7 for every $100 he lends him.

—Ed has $0. Ed is willing to lend to Denise, expecting no profit (but also no loss) for lending to her. Ed is also willing to lend to Grace, but wants a profit of $3

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for every $100 he lends her. Finally, Ed is willing to lend to Zoe; he expects no profit (but also no loss) for lending to Zoe, but he will lend at most $200 to Zoe.

—Grace has $0. Grace is willing to lend to Harry, but wants a profit of $2 for every $100 she lends him. Grace is also willing to lend to Zoe; she expects no profit for lending to Zoe, but she will lend at most $300 to Zoe.

—Harry has $0. Harry is willing to lend to Ed, expecting no profit (but also no loss) for lending to him. Harry is also willing to lend to Zoe; he expects no profit (but also no loss) for lending to Zoe, but he will lend at most $400 to Zoe.

What is the optimal (least expensive) way for Zoe to borrow her $1000? Note that you need to give not only the optimal solution, but also a (hopefully elegant) proof of why it is optimal. Also, for problems like this one, is it always the case that there is an optimal solution where the amount of money lent from one person to another is always a multiple of $100 (as long as the input numbers are all multiples of $100, as is the case here)?